Dust-lower-hybrid drift instabilities with dust charge fluctuations in an inhomogeneous dusty magnetoplasma

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Effects of a uniform magnetic field, the plasma inhomogeneity, and dust charge fluctuations on low-frequency dust-lower-hybrid drift waves have been investigated. Charging currents of electrons and ions to a spherical dust grain in a nonuniform magnetized dusty plasma have been calculated to study the charge fluctuation induced damping or growth of low-frequency drift waves. It is found that for strongly magnetized electrons and ions, the charge fluctuation damping is reduced significantly from that of an unmagnetized plasma. For sufficiently hot electrons, the drift wave exhibits instability in the absence of dust charge fluctuation damping.

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I. INTRODUCTION

Dusty plasmas are frequently found in space and laboratory environments where plasmas contain electrons, ions, and extremely massive, highly charged dust grains [1,2]. Waves and instabilities constitute the major part of basic research with emphasis on low-frequency electrostatic waves with and/or without the presence of a static magnetic field in dusty plasmas in recent years [3]. It has been shown that in the presence of low-frequency electrostatic waves, dust charge fluctuations lead to an additional damping mechanism in addition to Landau damping of these modes in a uniform, unmagnetized dusty plasma [4–6].

However, most laboratory and astrophysical dusty plasmas are almost invariably nonuniform in density and are confined by a homogeneous magnetic field either in ambient conditions, or is applied for convenience to control plasma confinement. Therefore, it is of much practical interest to make a rigorous investigation of the low-frequency waves and their instability in the presence of a uniform magnetic field in an inhomogeneous dusty plasma. Recently, the effect of an external uniform magnetic field on equilibrium charging currents to spherical dust grains and consequent dust charge fluctuation damping of the low-frequency dust-lower-hybrid waves has been examined in a homogeneous dusty plasma [7]. In this paper, we investigate the effects of the plasma inhomogeneity and a uniform magnetic field on the low-frequency dust-lower-hybrid waves and their instability (damping or growth) in a nonuniform dusty magnetoplasma.

In Sec. II, we derive the perturbed distribution function for plasma particles in the presence of low-frequency electrostatic waves in an inhomogeneous magnetized dusty plasma using the method of guiding center coordinates [8,9]. Expressions for dust charge fluctuations due to perturbed currents in the presence of electrostatic waves in a nonuniform dusty magnetoplasma are derived in Sec. III. Using the Poisson equation, including dust charge fluctuations, we then obtain in Sec. IV the dispersion relation which predicts the damping or the growth rate of electrostatic drift-kinetic waves for two important parameter regimes. The effects of the external magnetic field and the scale length of plasma inhomogeneity are also discussed in Sec. IV. Finally, a brief discussion of the results is presented in Sec. V.

II. LINEAR PERTURBED DISTRIBUTION FUNCTION

We consider small amplitude electrostatic waves propagating in a collisionless inhomogeneous plasma in a uniform magnetic field $\underline{B}_S \| \hat{z}$,

$$\Phi(\underline{x},t) = \phi_0 e^{-i(\omega t - \underline{k}\cdot\underline{x})}, \tag{1}$$

where ϕ_0 is the amplitude of the wave. The response of the plasma particles to this electrostatic perturbation can be described by the Vlasov equation,

$$\frac{\partial F_j}{\partial t} + (\underline{v}\cdot\underline{\nabla})F_j + \frac{q_j\underline{E}}{m_j}\cdot\underline{\nabla}_v F_j = 0, \tag{2}$$

where F_j is the total distribution function of the jth species, $\underline{E} = -\underline{\nabla}\Phi$, and q_j, m_j are the charge and mass of the jth species, respectively.

For convenience, we express the Vlasov equation in terms of the guiding center variables,

$$F(\underline{x},\underline{v},t) = F(\underline{x}_g,\mu,\theta,p_z,t), \tag{3}$$

where

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$$\underline{x}_g = \underline{x} - \frac{\underline{v} \times \underline{\omega}_{cj}}{\omega_{cj}^2}, \tag{4}$$

with $\omega_{cj}=q_jB_s/m_jc$, and

$$\left.\begin{aligned} \mu &= m_j v_\perp^2/2\omega_{cj}, \\ \theta &= \tan^{-1}(v_y/v_x), \\ p_z &= m_j v_z. \end{aligned}\right\} \tag{5}$$

The time-dependent velocity components can be described by

$$\dot{\underline{v}} = \frac{q_j\underline{E}}{m_j} + \underline{v} \times \underline{\omega}_{cj}, \tag{6}$$

where the overdot denotes the time derivative of the quantity involved.

Using the identity,

$$e^{-i(\omega t-\underline{k}\cdot\underline{x})} = e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)}\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}, \tag{7}$$

and Eqs. (4)–(6), one obtains [8,9]

$$\dot{x}_g = -\frac{iq_jk_y\phi_0}{m_j\omega_{cj}}\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)},$$

$$\dot{y}_g = \frac{iq_jk_x\phi_0}{m_j\omega_{cj}}\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)},$$

$$\dot{z}_g = \dot{z},$$

$$\dot{\mu} = -\frac{\partial H}{\partial\theta},$$

$$\dot{\theta} = \frac{\partial H}{\partial\mu},$$

$$\dot{p}_z = m\dot{v}_z = -iq_jk_z\phi_0\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)}, \tag{8}$$

where $H=\mu\omega_{cj}+p_z^2/2m_j+q_j\Phi$, $\rho=v_\perp/\omega_{cj}$, θ is the angle made by $v_\perp$ with the x axis, δ is the angle made by $\underline{k}_\perp$ with the x axis, and the symbol $\perp$ denotes a quantity perpendicular to the z axis.

The Vlasov equation in the guiding center coordinates can be written (omitting subscript j) as

$$\frac{\partial F}{\partial t} + \dot{\underline{x}}_g\frac{\partial F}{\partial \underline{x}_g} + \dot{p}_z\frac{\partial F}{\partial p_z} + \dot{\mu}\frac{\partial F}{\partial\mu} + \dot{\theta}\frac{\partial F}{\partial\theta} = 0. \tag{9}$$

In equilibrium, $\partial/\partial t=0$, $\underline{E}=0$, $\dot{\mu}=0$, $\dot{p}_z=0$, $\dot{\underline{x}}_g=0$, $\dot{\theta}=\omega_{cj}$, and the distribution function is a constant of motion,

$$F = f_0^0(\mu,p_z,x_g,y_g). \tag{10}$$

We consider only one-dimensional inhomogeneity in the equilibrium particle density,

$$n_0 = n_0^0\left(1+\frac{x}{L_n}\right), \tag{11}$$

where $L_n=-n_0^0/n_0'$ with $n_0'=\partial ln n_0(x)/\partial x$. The equilibrium distribution function can be taken as

$$f_0^0 = n_0^0\left(1+\frac{x_g}{L_n}\right)f_M(\mu,p_z), \tag{12}$$

where f_M can be taken as the Maxwellian,

$$f_M = \frac{1}{(2\pi)^{3/2}v_t^3}e^{-(\mu\omega_c+p_z^2/2m)/T}, \tag{13}$$

with $v_t=(T/m)^{1/2}$. Using Eq. (7), Eq. (1) can be written as

$$\Phi(\underline{x},t) = \phi_0e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)}\sum_n J_n(k_\perp\rho)e^{in(\theta-\delta)}, \tag{14}$$

and Eq. (9) can be written with $F=f_0^0+f$ as

$$\frac{\partial f}{\partial t} + v_z\frac{\partial f}{\partial z} + \dot{\theta}\frac{\partial f}{\partial\theta} + \dot{x}_g\frac{\partial f_0^0}{\partial x_g} + \dot{\mu}\frac{\partial f_0^0}{\partial\mu} + \dot{p}_z\frac{\partial f_0^0}{\partial p_z} = 0. \tag{15}$$

Expressing the perturbed distribution function in the presence of the potential Φ,

$$f = \sum_n f_ne^{in(\theta-\delta)}e^{-i(\omega t-\underline{k}\cdot\underline{x}_g)}, \tag{16}$$

we solve Eq. (15) to obtain

$$f_n = -\frac{n_0^0q\phi_0J_n(k_\perp\rho)}{\omega-k_zv_z-n\omega_c}\left[\left(1+\frac{x_g}{L_n}\right)\left(n\frac{\partial f_M}{\partial\mu}+\frac{k_z}{m}\frac{\partial f_M}{\partial v_z}\right) + \frac{k_yf_m}{m\omega_cL_n}\right]. \tag{17}$$

Thus

$$f = -n_0^0q\phi_0e^{-i(\omega t-\underline{k}\cdot\underline{x})}\sum_l\sum_n\frac{J_l(k_\perp\rho)J_n(k_\perp\rho)}{\omega-k_zv_z-n\omega_c}\left[\left(1+\frac{x_g}{L_n}\right)\times\left(n\frac{\partial f_M}{\partial\mu}+\frac{k_z}{m}\frac{\partial f_M}{\partial v_z}\right)+\frac{k_yf_m}{m\omega_cL_n}\right]e^{i(n-l)(\theta-\delta)}. \tag{18}$$

It is noticed from Eqs. (4) and (6) that the guiding center coordinates $\underline{x}_g$ are a function of the wave potential. Hence, finally, the linearized perturbed distribution function for the jth species in terms of the Maxwellian distribution function f_{Mj} is given by

$$f_j(\omega,\underline{k}) = -\frac{n_{0j}^0q_j\Phi(\omega,\underline{k})}{T_j}\sum_l\sum_n\left(1-\frac{\omega-\omega_j^*}{\omega-k_zv_z-n\omega_{cj}}\right)\times J_l(k_\perp\rho)J_n(k_\perp\rho)e^{i(n-l)(\theta-\delta)}f_{Mj}, \tag{19}$$

where $\omega_j^*=k_yT_j/m_j\omega_{cj}L_{nj}$ is the diamagnetic drift frequency and L_{nj} is the scale length of inhomogeneity of the jth species.

III. DUST CHARGE FLUCTUATIONS IN NONUNIFORM DUSTY MAGNETOPLASMAS

We consider electrostatic perturbations $(\omega,\mathbf{k})$ accounting for dust charge fluctuations. The charging equation for dust particles in a dusty plasma is

$$d_t Q_{d1} = I_{e1} + I_{i1}, \tag{20}$$

where Q_{d1} is perturbation of the dust charge in the presence of the perturbed electron and ion currents $I_{e1,i1}$ are associated with the perturbed plasma particle distribution functions in the electrostatic field of the low-frequency wave. Here, ω and $\mathbf{k}$ are the frequency and the wave vector, respectively.

To calculate the perturbed currents of magnetized electrons and ions in inhomogeneous plasma, we assume $\rho_{e,i} \gg a$, where a is the average radius of the dust grains and ρ_j is the Larmor radius of the species j (j is equal to e for electrons, i for ions and d for dust grains). We employ the guiding center coordinates [8,9] and obtain the perturbed distribution function in the presence of the electrostatic potential $\Phi(\mathbf{x},t)$, $f_{j1}(\mathbf{x},\mathbf{v},t)$ given by Eq. (19).

In the presence of low-frequency electrostatic fields, the charging current perturbations are

$$I_{j1}(\mathbf{x},t) \equiv \oint \mathbf{J}_j \cdot d\mathbf{S} = 2\pi a^2 q_j \int (v_\perp \cos\theta + v_\perp \sin\theta + v_\parallel) f_{j1} d\mathbf{v}, \tag{21}$$

where f_{j1} is given by Eq. (19). Using Eq. (21) and following Ref. [7], we then obtain, after straightforward calculation, the charging currents I_{j1} of electrons and ions $(j=e,i)$ in the inhomogeneous dusty magnetoplasma as

$$I_{j1}(\mathbf{x},t) = -4\pi a^2 q_j n_{j0} \frac{q_j \Phi(\mathbf{x},t)}{T_j} \exp\left(-\frac{q_j \Phi_G}{T_j}\right)\left(\frac{T_j}{2\pi m_j}\right)^{1/2} Y_j, \tag{22}$$

where

$$Y_j = Y_j^1 + Y_j^2 + Y_j^3, \tag{23}$$

with

$$Y_j^1 = \sqrt{\frac{\pi}{2}} \sum_n \frac{n\omega_{cj}}{k_\perp v_{tj}} I_n \exp(-b_j)\left[1 + \frac{\omega - \omega_j^*}{\sqrt{2} k_\parallel v_{tj}} Z(\xi_{nj})\right], \tag{24}$$

$$Y_j^2 = -i\sqrt{\frac{\pi}{2}} \frac{k_\perp v_{tj}}{\omega_{cj}} \sum_n \left[1 + \frac{\omega - \omega_j^*}{\sqrt{2} k_\parallel v_{tj}} Z(\xi_{nj})\right] \frac{d}{db_j}[I_n \exp(-b_j)], \tag{25}$$

$$Y_j^3 = \sqrt{\frac{\pi}{2}} \frac{\omega - \omega_j^*}{k_\parallel v_{tj}} \sum_n I_n \exp(-b_j)[1 + \xi_{nj} Z(\xi_{nj})], \tag{26}$$

and $\xi_{nj} = (\omega - n\omega_{cj})/\sqrt{2} k_\parallel v_{tj}$, $b_j = k_\perp^2 v_{tj}^2/\omega_{cj}^2$, with $v_{tj} = (T_j/m_j)^{1/2}$. In obtaining Eq. (22), we have assumed that the dust grain surface potential is constant. Here, $\Phi_G = Q_{d0}/a$ is the grain surface potential, and Q_{d0} is the equilibrium charge of a spherical dust grain of radius a.

It should be mentioned here that for a sufficiently large magnetic field, the electrons and ions of a plasma may be constrained to move in one dimension as if the gyrating particles are glued to the magnetic field. In this case where $\rho_{e,i} < a$, the shadow effect may be significant and the dust grains will not collect the same currents [10]. For a highly collisional unmagnetized plasma with small mean-free path λ_{mfp} compared to the intergrain distance r_0 $(\lambda_{\text{mfp}} \leq r_0)$, the shadow effect may also be neglected and the dust grains will collect the same currents. However, for a moderately strong magnetic field with $\rho_{e,i} \gg a$, the three-dimensional motion of the gyrating electrons and ions in velocity space must be taken into account with an appropriate distribution function. Our main aim in the present paper is to derive the distribution function by solving the Vlasov equation for an inhomogeneous magnetoplasma and calculate the charging currents. The results obtained are analyzed to find the instability or damping of very low-frequency electrostatic waves in the nonuniform dusty magnetoplasma.

IV. DISPERSION RELATIONS

We now consider two different parameter regimes of practical interest of our nonuniform dusty magnetoplasma to obtain the low-frequency dust-lower-hybrid drift wave dispersion relations and their damping or growth in Secs. IV A and IV B.

A. Strongly magnetized electrons and ions

For the low-frequency dust-lower-hybrid drift waves, we assume that the electrons and ions are strongly magnetized, and that the dust component is cold and unmagnetized:

$$\omega \ll \omega_{cj}, \quad j = e,i$$

$$k_\perp v_{tj} \ll \omega_{cj},$$

$$k_\parallel v_{tj} \ll |\omega|, \quad |\omega - n\omega_{cj}|. \tag{27}$$

Since the drift wave frequency is typically much larger than the dust plasma and dust gyrofrequencies, i.e., $\omega \gg \omega_{pd}, \omega_{cd}$, the dust grains can be considered immobile.

By integrating Eq. (19) within proper limits one can easily obtain the charge number density perturbation, $n_j(\omega,\mathbf{k}) \equiv -\chi_j k^2 \Phi(\omega,\mathbf{k})/4\pi q_j$. Thus, we calculate the electron and ion susceptibilities in view of the approximations in Eq. (27), and obtain

$$\chi_e = \frac{1}{k^2\lambda_{De}^2}\left[1 + \frac{\omega - \omega_e^*}{\sqrt{2} k_\parallel v_{te}} Z\left(\frac{\omega - n\omega_{ce}}{\sqrt{2} k_\parallel v_{te}}\right)(1 - b_e)\right], = \frac{\omega_e^*}{\omega k^2 \lambda_{De}^2} + \frac{\omega - \omega_e^*}{\omega}\left(\frac{k_\perp^2}{k^2}\frac{\omega_{pe}^2}{\omega_{ce}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pe}^2}{\omega^2}\right). \tag{28}$$

For $k_\parallel^2 \ll k_\perp^2$

$$\chi_e \simeq \frac{\omega_e^*}{\omega k^2 \lambda_{De}^2} + \frac{\omega - \omega_e^*}{\omega}\frac{\omega_{pe}^2}{\omega_{ce}^2}. \tag{29}$$

Similarly, for the ions we have

$$\chi_i \simeq \frac{\omega_i^*}{\omega k^2 \lambda_{Di}^2} + \frac{\omega - \omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2}. \tag{30a}$$

Neglecting the inhomogeneities and considering the cold magnetized electrons and ions and unmagnetized cold dust component, one readily obtains the usual dust-lower-hybrid wave [11] propagating nearly transverse to the external magnetic field direction from $\epsilon = 1 + \chi_e + \chi_i + \chi_d = 0$. We have

$$\omega^2 \approx \omega_{dlh}^2\left(1 + \frac{k_\parallel^2}{k_\perp^2}\frac{\omega_{pe}^2}{\omega_{pd}^2}\right), \tag{30b}$$

where $\omega_{dlh}^2 = \omega_{pd}^2\omega_{ci}^2/\omega_{pi}^2$, $\omega_{pi}^2 \gg \omega_{ci}^2$, and $k_\perp^2 \gg k_\parallel^2$.

We write $n_j = n_{j0} + n_{j1}$ and $Q_d = Q_{d0} + Q_{d1}$ where the quantities with subscript 1 denote perturbed quantities due to the presence of any low-frequency wave $(\omega, \mathbf{k})$. Thus, from Eq. (20), we obtain, following Varma *et al.* [4],

$$Q_{d1} = -i\beta(\omega)\Phi/\omega, \tag{31}$$

where

$$\beta(\omega) = \frac{a^2}{\sqrt{2\pi}}\left[\frac{\omega_{pe}}{\lambda_{De}}\exp\left(\frac{e\Phi_G}{T_e}\right)Y_e(\omega) + \frac{\omega_{pi}}{\lambda_{Di}}\left(1 - \frac{e\Phi_G}{T_i}\right)Y_i(\omega)\right]. \tag{32}$$

Here, $\lambda_{Dj} = v_{tj}/\omega_{pj}$, $\omega_{pj} = (4\pi n_{j0} q_j^2/m_j)^{1/2}$, and $Y_{e,i}(\omega)$ is given by Eq. (23) depending on conditions of the wave perturbation and plasma parameters. For the conditions here in Sec. IV, e.g., in Eq. (27), we have

$$Y_j \simeq \sqrt{\frac{\pi}{2}}(1 - b_j)\left[\frac{k_\perp v_{tj}}{\omega_{cj}}\left\{\frac{\omega - \omega_j^*}{\omega_{cj}} + i\left(1 - \frac{\omega - \omega_j^*}{\omega}\left(1 + \frac{k_\parallel^2 v_{tj}^2}{\omega^2}\right)\right)\right\} - \frac{\omega - \omega_j^*}{\omega}\frac{k_\parallel v_{tj}}{\omega}\right]. \tag{33}$$

If we assume $k_\parallel/k_\perp \ll \omega^2/\omega_{cj}^2$ and neglect the small imaginary term, the factor Y_j $(j = e, i)$ that modify the charge fluctuation factor β of the magnetized plasma turns out to be

$$Y_j \approx \sqrt{\frac{\pi}{2}}(1 - b_j)\frac{k_\perp v_{tj}}{\omega_{cj}}\frac{\omega - \omega_j^*}{\omega_{cj}}. \tag{34}$$

Let us now demonstrate how the external magnetic field and the scale length of plasma inhomogeneity affect the damping of a drift-kinetic wave propagating perpendicular to the magnetic field direction. For ω_{cd}, kv_{td}, $k_\parallel v_{ti}$, $k_\parallel v_{te} \ll |\omega| \ll \omega_{ci} \ll \omega_{ce}$, $|\omega| \gg \omega_{pd}$, ω_{cd} and $b_{e,i} \ll 1$, the electrons and ions are strongly magnetized, while the cold and unmagnetized dust grains are considered immobile.

The Poisson equation in the presence of a low-frequency mode and dust charge fluctuations is

$$k^2\Phi + 4\pi(n_{e1}e - n_{i1}e - n_{d1}Q_{d0} - n_{d0}Q_{d1}) = 0, \tag{35}$$

where Q_{d1} is given by Eq. (31). Inserting $n_{e1,i1} = \pm k^2\chi_{e,i}\Phi/4\pi e$, $n_{d1} = 0$, and Q_{d1} given by Eq. (31), we obtain $\epsilon(\omega, \mathbf{k})\Phi(\omega, \mathbf{k}) = 0$, where

$$\begin{aligned}\epsilon(\omega,\mathbf{k}) &= 1 + \chi_e + \chi_i + \frac{i4\pi n_{d0}\beta}{k^2\omega}\\ &= 1 + \frac{\omega_e^*}{k^2\lambda_{De}^2\omega} + \frac{\omega_i^*}{k^2\lambda_{Di}^2\omega} + \frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1 + \frac{n_{e0}m_e}{n_{i0}m_i}\right) - \frac{\omega_e^*}{\omega}\frac{\omega_{pe}^2}{\omega_{ce}^2}\\ &\quad - \frac{\omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2} + i\frac{4\pi n_{d0}\beta}{k^2\omega}.\\ &\simeq 1 + \frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{\omega_i^*}{\omega}\frac{\omega_{pi}^2}{\omega_{ci}^2}\left(1 + \frac{T_e}{T_i}\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\frac{L_{ni}}{L_{ne}}\right) + i\frac{4\pi n_{d0}\beta}{k^2\omega}.\end{aligned} \tag{36}$$

In the above, $k_\parallel^2 \ll k_\parallel^2$ and $\omega_{pe,i}^2/\omega_{ce,i}^2 \gg 1/k^2\lambda_{De,i}^2$ have been assumed. By equating the real and imaginary parts from $\epsilon = 0$, we obtain the dispersion relation for the dust-lower-hybrid drift waves as (with $\omega \equiv \omega_R + i\gamma$)

$$\omega_R = \omega_i^*\left(1 + \frac{T_e}{T_i}\frac{m_e}{m_i}\frac{n_{e0}}{n_{i0}}\frac{L_{ni}}{L_{ne}}\right), \tag{37}$$

and the damping rate of this drift wave due to the dust charge fluctuation as

$$\gamma = -\frac{4\pi n_{d0}\beta}{k^2(\omega_{pi}^2/\omega_{ci}^2)}, \tag{38}$$

for $\omega_{pi}^2 \gg \omega_{ci}^2$. Comparing this with Eq. (20) of Ref. [7] we find that the dust charge fluctuation damping of the low-frequency drift wave is less than that of the usual dust-lower-hybrid wave or the dust-acoustic wave by a factor $\omega_{pi}^2/\omega_{ci}^2$, which is greater than 1. Hence, the low-frequency dust-lower-hybrid drift wave is more stable than the usual dust-acoustic and dust-lower-hybrid waves.

B. Hot electrons

Here, we study the dust-lower-hybrid drift instability in the presence of dust-charge fluctuation damping in a nonuniform magnetized dusty plasma where the electrons are thermal, the ions are strongly magnetized, and the dust grains are immobile (cold and unmagnetized) as $\omega \gg \omega_{pd}, \omega_{cd}$. Consequently, the following conditions are valid for this case:

$$\begin{gathered}\omega_{cd} \ll \omega \ll \omega_{ci} \ll \omega_{ce},\\ k_\parallel v_{te} \gg |\omega|,\ |\omega - \omega_e^*|,\ |\omega - n\omega_{ce}|,\\ k_\parallel v_{ti} \ll |\omega|,\ |\omega - n\omega_{ci}|,\\ b_{e,i} \ll 1.\end{gathered} \tag{39}$$

Thus, we have

$$\chi_e = \frac{1}{k^2\lambda_{De}^2}\left[1 + i\sqrt{\pi}\left(\frac{\omega - \omega_e^*}{\sqrt{2}k_\parallel v_{te}}\right)\right],$$

$$\chi_i = \frac{1}{k^2\lambda_{Di}^2}\frac{\omega_i^*}{\omega} + \frac{\omega - \omega_i^*}{\omega}\left(\frac{k_\perp}{k^2}\frac{\omega_{pi}^2}{\omega_{ci}^2} - \frac{k_\parallel^2}{k^2}\frac{\omega_{pi}^2}{\omega^2}\right), \qquad (40)$$

$$\chi_d = 0.$$

Using the conditions given by Eqs. (39) and following the procedures in Sec. IV A, we obtain the dispersion relation for the dust-lower-hybrid drift wave and the damping rate in the presence of charge fluctuation damping ($\omega \equiv \omega_R + i\gamma$)

$$\omega_R = -\frac{n_{i0}}{n_{e0}}\frac{T_e}{T_i}\frac{\omega_i^*}{1 + k_\perp^2\rho_s^2}, \qquad (41)$$

$$\gamma = \left[\sqrt{\frac{\pi}{2}}\frac{\omega_R}{k_\parallel v_{te}}\frac{k_y c k_B T_e}{n_{e0}e^2 B_0}\frac{\partial}{\partial x}(q_{d0}n_{d0}) - 4\pi n_{d0}\beta\lambda_{De}^2\right], \qquad (42)$$

where $k_\perp^2\rho_s^2 \ll 1$ with $\rho_s = \omega_{pi}\lambda_{De}/\omega_{ci}$. For this case, Eqs. (39), the modification factors in $\beta(\omega)$, given by Eq. (33), upon simplification reduce to

$$Y_e \simeq \sqrt{\frac{\pi}{2}}(1 - b_e)\left(\frac{\omega - \omega_e^*}{k_\parallel v_{te}} + i\frac{k_\perp v_{te}}{\omega_{ce}}\right), \qquad (43)$$

$$Y_i \simeq \sqrt{\frac{\pi}{2}}\frac{k_\perp v_{ti}}{\omega_{ci}}\frac{\omega - \omega_i^*}{\omega_{ci}}(1 - b_i). \qquad (44)$$

Thus, $Y_e \approx \sqrt{\pi/2}\,\omega_R/k_\parallel v_{te}$ and $Y_i \approx \sqrt{\pi/2}(k_\perp v_{ti}/\omega_{ci})(\omega_R/\omega_{ci})$ are less than unity, thereby reducing the charge fluctuation damping in the magnetized plasma [cf. Eq. (32)].

We note from Eq. (42) that when the charge fluctuation damping is negligible, the drift wave grows if $\partial(q_{d0}n_{d0})/\partial x > 0$. This instability is due to motion of the electrons in the presence of the drift wave.

V. DISCUSSION

We have investigated the effects of an external uniform magnetic field and the plasma inhomogeneity on low-frequency dust-lower-hybrid drift waves in the presence of dust charge fluctuations in a nonuniform dusty magnetoplasma. In the presence of low-frequency electrostatic waves, the dust charging current perturbations were calculated explicitly to examine the dust charge fluctuation effects. To calculate the dust charging currents, we employed the method of guiding center coordinates to solve the Vlasov equation and obtained the perturbed distribution function in the presence of low-frequency drift waves. We assumed spherical dust grains and obtained the dust charge fluctuations. Effects of a homogeneous magnetic field and the dust density inhomogeneity were included in the dispersion relation and damping or growth of the drift waves was investigated. It was found that for strongly magnetized electrons and ions, the charge fluctuation induced damping is reduced significantly from that of an unmagnetized plasma [cf. Eq. (38)]. Thus, it was noticed that low-frequency dust-lower-hybrid drift waves are relatively more stable than usual dust-acoustic and dust-lower-hybrid waves. For sufficiently hot electrons, the drift wave exhibits instability in the absence of charge fluctuation damping [cf. Eq. (42)].

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